Identification of a Two Stage Active Seismic Isolation Platform for Laser Interferometer Gravitational-Wave Observatory

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ME 323: Modeling and Identification of Mechanical Systems for Control
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1 Introduction

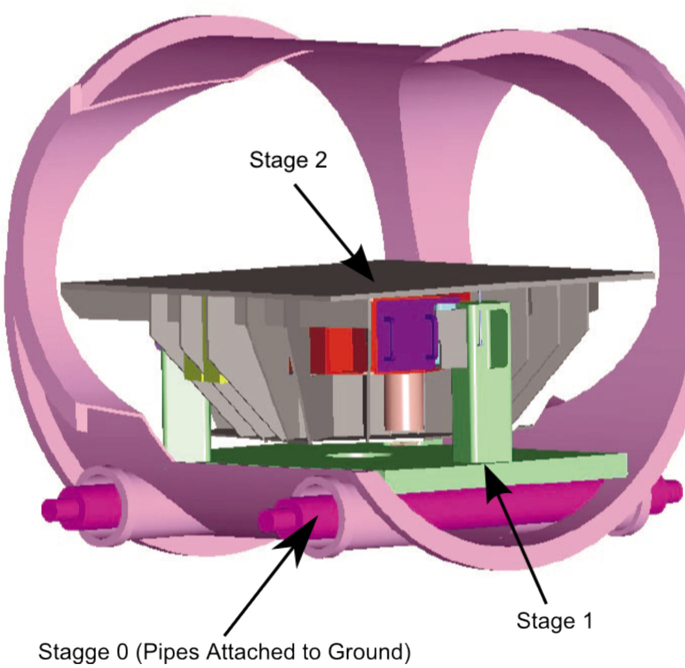

Figure 1: LIGO Seismic Isolation Platform

1.1 System Description

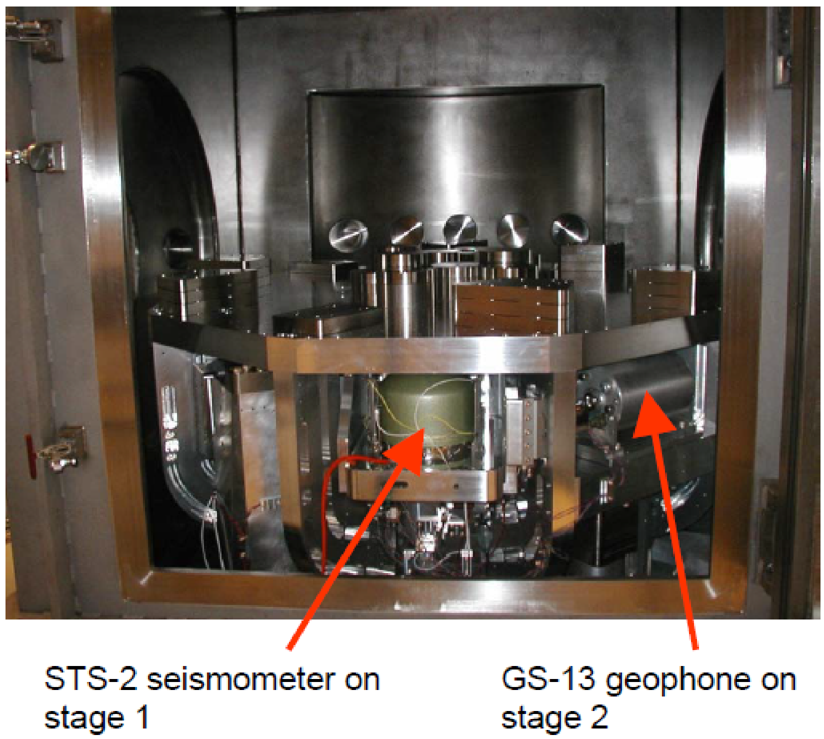

Figure 2: LIGO Sesmic Isolation Table (shown with sensors)

The system examined was a two stage active seismic isolation platform used for the Laser Interferometer Gravitational-Wave Observatory (LIGO). The platform will be used in the next generation gravity wave detection observatory, Advanced LIGO. The isolation platform is used to isolate the sesmic table from environmental disturbances. The platform isolates the optics attached to the table from external distrurbances and allows for extremely precise measurements.

The isolation platform consists of Three Stages: Stage 0, Stage 1 and Stage 2, as shown in Figure 1. Stage 0 consists of two large pipes that are fixed to the ground. In this analysis, we assume Stage 0 is rigid. Stage 1 is attached to Stage 0 through a series of stiff blade springs and short pendulum links [1]. Stage 2 is attached to Stage 1 in the same way. The system has electromagnetic non-contacting forcers to actuate movement between Stage 0 and Stage 1, and Stage 1 and Stage 2.

1.2 Sensor Description

Stage 1 has 6 capacitive position sensors, 3 Streckeisen STS2 seismometers, and 6 Mark L4C geophones. Stage 2 has 6 capacitive position sensors and 6 GeoTech GS13 seismometers. The capacitive stage sensors are best from DC to a few Hz, while the geophones are best from 1 Hz to 1 kHz. The seismometers read well at medium frequencies from 1 Hz to 200 Hz. See Figure 2 for an image of the sensors.

The capacitive position sensors are displacement sensors - their output corresponds to the absolute displacement between two stages. The seismometers and geophones are inertial sensors, so their output corresponds to the velocity in inertial space.

2 Physical Model

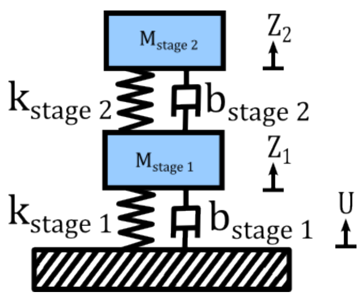

Figure 3: LIGO Sesmic Isolation Platform Abstracted as a Coupled Mass-Spring-Damper System

Since we assume Stage 0 is rigidly attached to the ground, we have a coupled mass-spring-damper system. If we constrain the stage movement to the vertical direction, we have the system shown in Figure 3. Note that we model an input displacement (u) as ground displacement.

The governing equations LIGO Sesmic Isolation Platform Modeled as a coupled mass-spring damper system are:

$$m_1\ddot{z_1} = k_1(u - z_1) + k_2(z_2 - z_1) + b1(\dot{u} - \dot{z_1}) + b_1(\dot{u} - \dot{z_1}) + b_2(\dot{z_2} - \dot{z_1})$$

$$m_2\ddot{z_2} = k_2(z_1 - y) + b_2(\dot{z_1} - \dot{y})$$

In State Space form, the equations can be represented as:

$$\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \begin{pmatrix} \ddot{z_1} \\ \ddot{y} \end{pmatrix} + \begin{pmatrix} b_1 + b_2 & -b_2 \\ -b_2 & b_2 \end{pmatrix} \begin{pmatrix} \dot{z_1} \\ \dot{y} \end{pmatrix} + \begin{pmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} k_1 \\ 0 \end{pmatrix} u + \begin{pmatrix} b_1 \\ 0 \end{pmatrix} \dot{u}$$

Taking the Laplace Transforms of the state space form yields:

$$\begin{pmatrix} m_1 s^2 + (b_1 + b_2)s + (k_1 + k_2) & -b_2 s - k_2 \\ -b_2 s - k_2 & m_2 s^2 + b_2 s + k_2 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} k_1 + b_1 s \\ 0 \end{pmatrix} u$$

Solving for the transfer function from the input to the displacement of the first stage yields:

$$\frac{Z_1(s)}{U(s)} = \frac{(m_2s^2 + b_2s + k_2)(k_1 + b_1s)}{(m_1m_2)s^4 + [m_2(b_1 + b_2) + m_1b_2]s^3 + [m_2(k_1 + k_2) + m_1k_2 + b_2b_1]s^2 + [k_2b_1 + b_2k_1]s + [k_1k_2]}$$

Solving for the transfer unction from the input to the displacement of the second state yields:

$$\frac{Z_2(s)}{U(s)} = \frac{(-b_2s - k_2)(k_1 + b_1s)}{(m_1m_2)s^4 + [m_2(b_1 + b_2) + m_1b_2]s^3 + [m_2(k_1 + k_2) + m_1k_2 + b_2b_1]s^2 + [k_2b_1 + b_2k_1]s + [k_1k_2]}$$

With the current experimental setup, we cannot measure these parameters, so to identify the system we will rely on frequency and time domain identification methods.

3 Frequency Domain Identification

3.1 ETFE of Stage 1

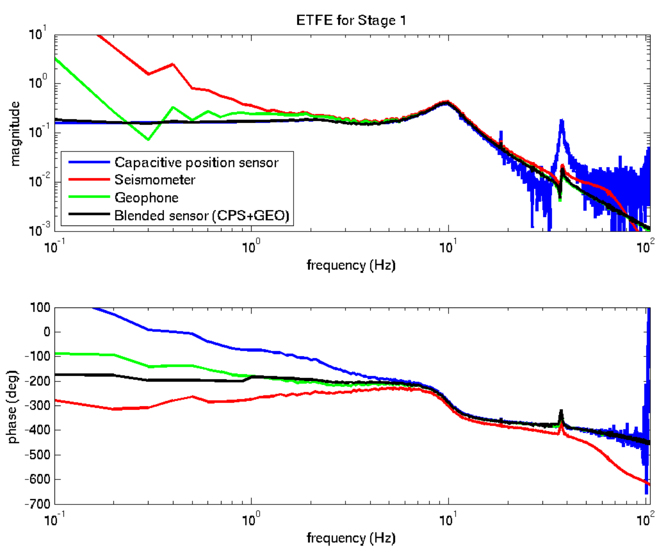

Figure 4: ETFE of Stage 1 of the Sesmic Isolation Platform

The empirical transfer function estimate (ETFE) of stage 1 was obtained for each of the three sensors (capacitive seismometer, geophone) located on stage 1. We used a 10s chirp from 100mHz to 100Hz to as our driving input. To remove the effect of transients, two chirps were inputted back to back, and only the input and output data arising from the second chirp was considered. Note that since the displacement and inertial sensors are outputting fundamentally different quantities, the ETFEs for the inertial sensors had to be convolved with a transfer function that mapped inertial velocities to displacements. The resulting ETFEs are shown in Figure 4.

From our physical modeling, we expect to see 3 zeros and 4 poles in the ETFE. There is a clear resonance around 10Hz, which accounts for two poles. There is a near pole-zero cancelation near 37 Hz, but this is not a resonance we can trace back to our physical model. It's actually caused by a vibrational mode of the pipes in Stage 0 whenever we actuate motion between Stage 0 and Stage 1. Since we assumed Stage 0 was rigid, this mode does not show up in the physical model derivation. The final resonance of the system should be around the natural frequency of the platform, around a few Hz. However, when taking our data the active damping loops were turned on, which almost completely damped the low frequency mode. With the loops off, on the other hand, the sensors would have saturated, so this was a necesarry tradeoff.

3.1.1 Sensor Blending

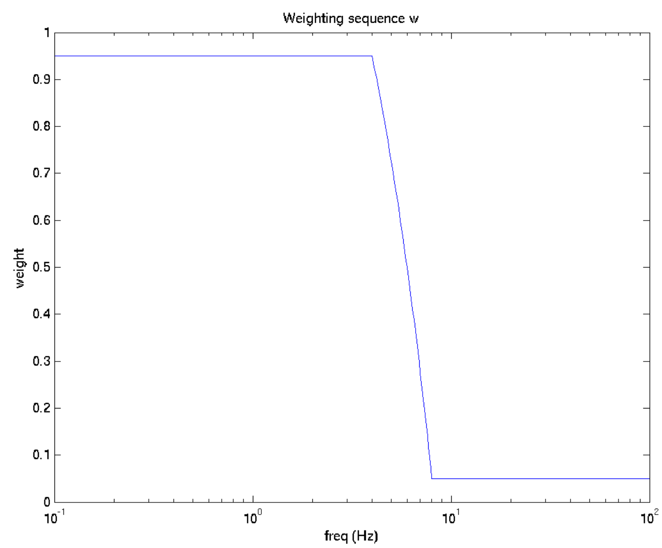

Figure 5: Weighting Sequence for Sensor Blending Algorithm

Since our sensors perform best in different frequency ranges, a better approximation of the ETFE can be obtained by using information from all sensors simultaneously. The idea is to weight the output information of a sensor more heavily in the frequencies that it is most sensitive to. We blended only two sensors (geophone and the capacitive displacement sensor) because we found the geophone and seismometer provide nearly identical information across the same frequency band. The weight algorithm is a weighted geometric mean of the ETFEs based on a desired weighting sequence. We designed a weighting sequence that took 95% of capacitive ETFE below 4 Hz, 5% of the capacitive ETFE above 8 Hz and a linear ramp in between, as shown in Figure 5. Since the two transfer functions were close in magnitude at that frequency, we had a fairly smooth transition:

$$ETFE_{Blended} = ETFE_1{}^{w(f)} ETFE_2{}^{1-w(f)}$$

3.2 ETFE of Stage 2

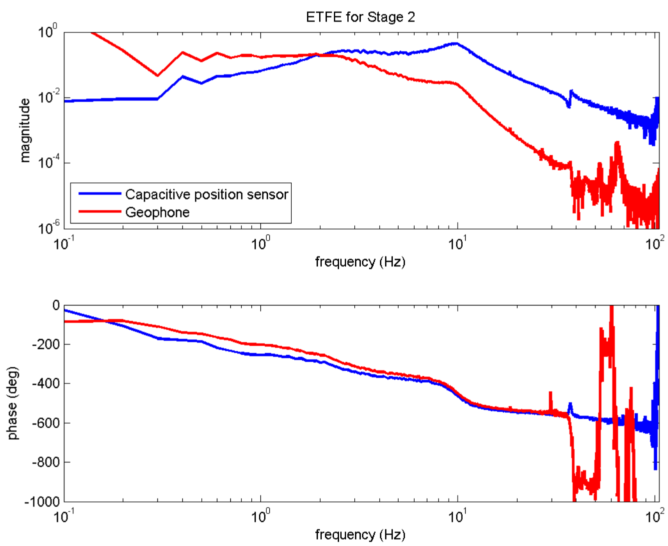

Figure 6: ETFE of Stage 2 of the Sesmic Isolation Platform

To obtain an ETFE from a ground input displacement to stage 2, we used the previous chirp input and examined the output of the geophone and capacitive position sensor mounted on stage 2. The resulting ETFE is shown in Figure 6. The two sensors agree well in phase up to about 30 Hz, but otherwise our sensor readings are unreliable.

4 Time Domain Identification

4.1 Model Creation

For the time domain identification, a model of the system outputs with respect to the displacement input needs to be created. From our physical model, we

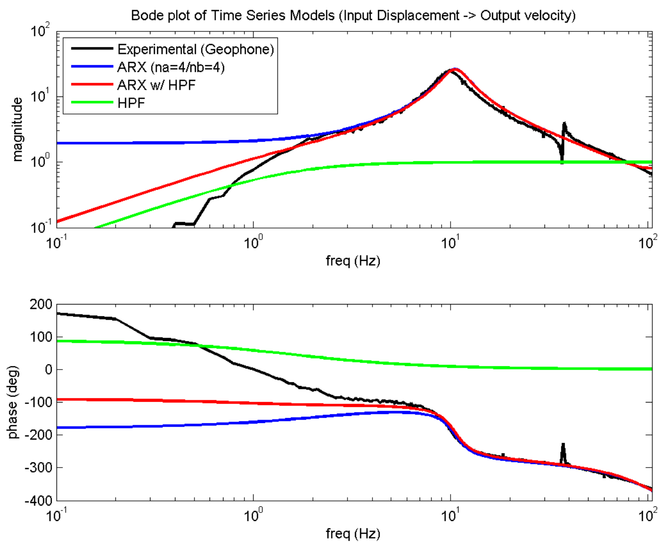

Figure 7: Bode Plot Derived from an ARX Model with Schroeder Phase Input

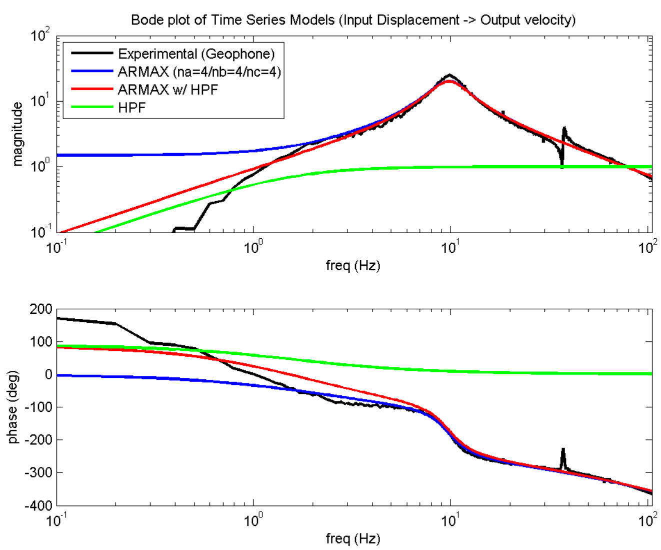

Figure 8: Bode Plot Derived from an ARMAX Model with Schroeder Phase Input

know our continuous transfer function $\frac{Z_1(s)}{U(s)}$ should have numerator order 3 and denominator order 4. Discretizing via Tustin, we get a model that is 4th order in the numerator and denominator.

Since our displacement and inertial sensors output fundamentally different quantities, we will use the geophone to identify the system (since it is valid over almost the full frequency range of interest). The transfer function we will model, therefore, is Velocity of Stage 1 in response to a Displacement input, or $\frac{\dot{Z_1}(q)}{U(q)}$.

To generate a parametric model, we require the time series of input and output. To generate a time series input, we chose to use a sum of sinusoids. We wanted to have sinusoidal inputs across a wide range of frequencies, while keeping the crest factor of our input signal as low as possible. To create the signal, we used the Schroeder phase algorithm given in the lecture notes:

$$u(t) = \sum_{k=1}^{N} \cos w_k t + \phi_k$$

where

$$w_k = w_0 + \alpha k$$

and

$$\phi_k = \frac{k(k-1)}{n}\pi$$

We chose to use N = 100 sine waves, equally spaced from 1-100Hz.

We considered ARX and ARMAX models only, for simplicity. The Bode plot of our parameterized ARX model is shown in Figure 7 and the ARMAX model is shown in Figure 8. Note that since we used the geophone, an inertial velocity sensor not valid at low frequency, to extract the time series data, we did not roll off at DC. Therefore, we added high pass filters to the ARX and ARMAX models in order to get the proper low frequency behavior (in particular, DC gain).

The ARX models successfully capture the peak at 10Hz, with some slight offset. They don't capture the Stage 0 kink around 37Hz. Instead, the model uses it's degrees of freedom to fit to a rolled off higher frequency peak (not shown). The ARMAX model is similar, except it more closely matches the 10Hz peak, and also more closely follows the experimental transfer function above 10Hz.

4.2 Model Validation

We validate the emprically determined model by overlaying a step response of the actual system against the simulated step response of the emprically determined model. In both the 4th order ARX and ARMAX based models, the models have some steady state error relative to experimental step response (see Figure 9. We expect the steady-state velocity response to a input step displacement to be zero. However, as mentioned earlier, the geophone is not capable of resolving DC gain. Overall, the step response matches fairly well in phase, and

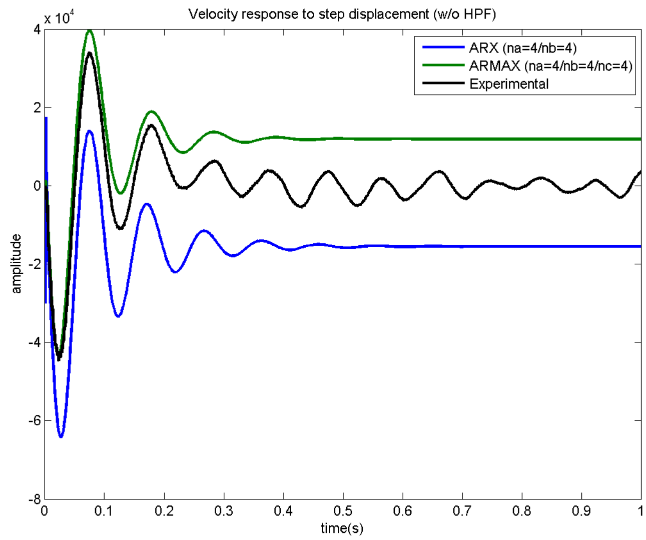

Figure 9: Step Response Validation for Time Domain Identification

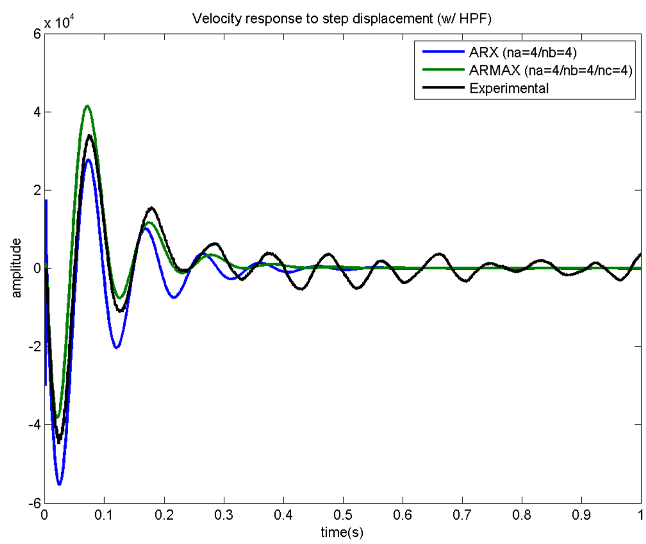

Figure 10: Step Response Validation after Applying High Pass Filter

almost as well in magnitude (except for the SS error). So our 4th order model seems to be a good starting point.

If we examine the step response of our parametric models with high pass filter, we see that the steady state error is eliminated, as expected (see Figure 9. Now there is good agreement in magnitude, phase, and steady state value. Since the geophone is noisy, it never converges to a steady state value, although it clearly oscillates about zero, as expected. It also appears that our ARMAX model is a slightly better fit to the experimental step response in terms of peak magnitude and phase, so we have some minor noise coloration.

5 Conclusion

Using a 4^{th} order coupled mass-spring based ARMAX model, we were able to successfully capture the system dynamics in the desired frequency range of 100mHz to 100Hz. Although we could not capture the low frequency resonance due to damping loops being on, the model's step response was close to the experimental response.

References

[1] N A Robertson et al, *Seismic isolation and suspension systems for Advanced LIGO*. "Gravitational Wave and ParticleAstrophysics Detectors", Proceedings of SPIE, vol. 5500, 2004.